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## COMMENT

### A COMMENT ON THE SUPPOSED PARADOXES OF A MATHEMATICAL INTERPRETATION OF THE LOGIC OF TRIALS†

DALE A. NANCE\*

This symposium has renewed discussion of the apparent paradox of attempting to apply mathematical probabilities to the logic of trials. Brought to academic attention by the stimulating work of L. Jonathan Cohen,<sup>1</sup> the debate reflects a more general controversy over the virtues of quantifying legal decisionmaking. In this brief comment I do not intend to take sides either in this debate or in the more general controversy. I want only to draw attention to a weakness in the legal interpretations which underlie much of the debate.

The paradoxes that Cohen identified can, for the most part, be subsumed under the following proposition: If one hypothesizes that the rules applicable to mathematical probability<sup>2</sup> apply to the logic of legal inference, then one is led to prescribe results for identifiable situations that are inconsistent with the established legal rules governing them. Thus, the hypothesis that mathematical probability applies in this way must be mistaken, at least in some contexts and as a descriptive matter. Cohen then gives an account of probabilistic reasoning that he claims is more compatible with juristic reasoning.

There are three distinguishable steps in the kind of argument that Cohen has employed. First, one must accurately identify the prescribed results of applying mathematical probability to the designated situations. Second, one must accurately identify the established legal rules on the relevant issues. Finally, of course, one must compare the identified results with the legal rules, and show the two to be inconsistent. I believe that at least the second step in this process has been defectively performed with respect to each supposedly paradoxical situation that Cohen identifies. Since the second step in the analysis is essentially a matter of legal research and interpretation, it is perhaps not surprising that Cohen, who is not a lawyer, may have been misled by the legal authorities upon which he has relied. It is regretta-

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<sup>1</sup> L. COHEN, *THE PROBABLE AND THE PROVABLE* (1977).

<sup>2</sup> It is clear from the debates that what is meant by "mathematical probability" is a system of probability measurement that satisfies the Kolmogorov axioms. See, e.g., *id.* at § 2; Kaye, *Paradoxes, Gedanken Experiments and the Burden of Proof: A Response to Dr. Cohen's Reply*, 1981 ARIZ. ST. L.J. 635, 636-41. See generally T. FINE, *THEORIES OF PROBABILITY* 58-60 (1973).

ble, however, that academic lawyers who have joined in the debate have failed to utilize their special skills to investigate the purely legal issues more thoroughly.

Perhaps my point can be seen best in the context of the supposed difficulty about conjunction.<sup>3</sup> In its simplest form, the claim of paradox here runs as follows.<sup>4</sup> Assume a simple civil case in which there are two essential elements in the plaintiff's cause of action, A and B, and no complicating affirmative defenses or counterclaims. The proper mathematical interpretation of the preponderance of the evidence standard in civil cases, we are told, is that the probability of the event to be proved must be greater than .5. Therefore, a mathematical interpretation of the plaintiff's burden is that the plaintiff should win if, and only if, the probability that both A and B are true is greater than .5. That is to say, if Pr is the probability function, which is taken to satisfy the mathematical calculus of chances:

P wins if and only if  $\text{Pr}(A \text{ and } B)$  is greater than .5.  
(Criterion I)

However, we are told that this is not equivalent to the established legal standard. Rather, we are told, the legal standard is that the plaintiff wins if and only if *each* element is severally proved by a preponderance of the evidence. Accepting the mathematical interpretation of the preponderance of the evidence standard, this would have to mean:

P wins if and only if  $\text{Pr}(A)$  is greater than .5  
and  $\text{Pr}(B)$  is greater than .5.  
(Criterion II)

It is clear that (I) and (II) are not equivalent criteria, and the hypothesis has generated a contradiction. A typical, though obviously unrealistic, example of the difference between the two standards is to assume A and B are probabilistically independent and that  $\text{Pr}(A) = .6$  and  $\text{Pr}(B) = .6$ . Then, under the multiplicative rule of mathematical probabilities,  $\text{Pr}(A \text{ and } B) =$

<sup>3</sup> A few comments along the lines of the point made here have appeared in this symposium regarding another supposed paradox, that concerning the mathematical analysis of negation, as it arises in the context of the gatecrasher problem. See Allen, *A Reconceptualization of Civil Trials*, 66 B.U.L. REV. 401, 405-06 (1986); see also *Kaminsky v. Hertz Corp.*, 288 N.W.2d 426 (Mich. App. 1979). With regard to the problem of catenated inferences, see J. WEINSTEIN, J. MANSFIELD, N. ABRAMS, M. BERGER, *CASES AND MATERIALS ON EVIDENCE* 32-35 n.5 (7th ed. 1983).

<sup>4</sup> See Cohen, *supra* note 1, at ch. 5. Actually, Cohen considers several possible mathematical interpretations of the civil standard of proof, but in each case—including the main one discussed here—he claims that the mathematician is embarrassed by the consequences of his approach. One important possibility that he does not consider is that "preponderance" might have a context-dependent, but nonetheless quantitative, measure.

$\Pr(A) \times \Pr(B) = .36$ , so that the plaintiff loses under Criterion I but wins under Criterion II. More realistic assumptions can generate similar problems, or at least so it is claimed.

The problem I want to point out concerns the claim that the established legal standard is the plaintiff wins if and only if each element is severally proved by a preponderance of the evidence. In particular, the issue is the accuracy of the claim that, in the determination of facts, the law concerns itself only with the probabilistic evaluation of the elements *seriatim*, and not with the probabilistic evaluation of their conjunction.<sup>5</sup> I have not undertaken an exhaustive study of the authoritative interpretations of the burden of persuasion in this regard, but what I have found is peculiarly unhelpful. Cohen himself simply says, without citing legal authority, that "[t]he rule for civil suits requires a plaintiff to prove each element of his case on the balance of probability," and that a conjunction rule, with its multiplicative consequence under mathematical probabilities, "seems to be a rule that is unknown to judges and unrespected by triers of fact."<sup>6</sup>

One obvious place to look for guidance on this issue is jury instructions. A typical example, cited by one writer who accepts that there is a paradox here,<sup>7</sup> is the following instruction patterned for use in federal civil cases:

The burden is on the plaintiff in a civil action, such as this, to prove every essential element of his claim by a preponderance of the evidence. If the proof should fail to establish any essential element of plaintiff's claim by a preponderance of the evidence in the case, the jury should find for the defendant.<sup>8</sup>

This instruction is subject to a variety of interpretations. In particular, lawyers (reputation notwithstanding) are sloppy enough in the use of language that "every" in the first sentence could mean "all" or it could mean "each."<sup>9</sup> The former interpretation makes more palpable an affinity to Criterion I; the latter looks more like Criterion II. The ultimate difficulty, however, is that, whichever word one substitutes, the denotation of conjunc-

<sup>5</sup> Cohen's explanation would be that because the law embraces his system of "inductive" probability it ought not to concern itself directly with a conjunctive probability. Under Cohen's system, the probability of the conjunction of two events either is meaningless or, where meaningful, is equal to the smallest of the probabilities of the individual elements. In the latter case, an inductive probability version of Criterion I is equivalent to an inductive probability version of Criterion II, and contradiction is thus avoided. See COHEN, *supra* note 1, at § 71.

<sup>6</sup> *Id.* at 58, 59. Elsewhere Cohen provides numerous citations to legal authorities, see *id.* at ch. 4 (entitled "What are the Standards of Proof in Courts of Law?"), but none specifically on the point of the conjunction problem or the multiplication rule.

<sup>7</sup> Allen, *supra* note 3, at 405.

<sup>8</sup> E. DEVITT & C. BLACKMAR, FEDERAL JURY PRACTICE AND INSTRUCTIONS § 71.14 (3d ed. 1977).

<sup>9</sup> See, e.g., BLACK'S LAW DICTIONARY 498 (5th ed. 1979).

tion in these terms is inexactly specified. It is not clear where the "and" is to be placed, within the functional parentheses, as in Criterion I, or outside of them, as in Criterion II.

In either case, it seems to be contemplated in the model instruction that the first sentence logically entails the second, and that the second separately states certain necessary conditions for the plaintiff to win. As the second sentence contains (logicians would say) no new information, the purpose of the second sentence seems to be simply to remind the jury that a failure by the plaintiff to sufficiently prove any element of the plaintiff's case will relieve the jury of further deliberations on the other elements.<sup>10</sup> But is such an interpretation of the second sentence really consistent with the mathematicist's preferred Criterion I? The answer is yes, for the simple reason that under mathematical probabilities, the probability of any conjunctive event can be no greater than that of any constituent event by itself. If, therefore,  $\text{Pr}(A)$  were to be estimated with reasonable confidence by the jury as less than .5, it would necessarily follow under a mathematicist interpretation that  $\text{Pr}(A \text{ and } B)$  is less than .5, and the defendant should win without the necessity of considering or determining  $\text{Pr}(B)$ .<sup>11</sup> What does not follow, of course, is that the jury is properly instructed to give a verdict for the plaintiff if they find *only* that the probability of each element in the plaintiff's claim exceeds .5. Stated formally, the simultaneous satisfaction of each condition in a set of necessary conditions is not necessarily a sufficient condition. It may not be too surprising, however, that some people talk as if it were. Indeed, even knowledgeable participants in this debate argue that the specification of necessary conditions in such a jury instruction is inconsistent with a mathematicist interpretation: they infer that the collective satisfaction of each of the necessary conditions should be sufficient to give the verdict for the plaintiff.<sup>12</sup>

Obviously, such a simple analysis is just the beginning of a thorough interpretation of provisions like the quoted jury instruction. Much more can and should be said by anyone claiming that there is an established legal rule as to the exact application of the civil standard to multiple elements of a claim. (I would suggest some serious research into the use of special verdicts and interrogatories to the jury.) Here, I will only indicate a little further just how a mathematicist interpretation is reasonably plausible in this context.<sup>13</sup>

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<sup>10</sup> This interpretation is confirmed by other jury instructions. See, e.g., E. DEVITT & C. BLACKMAR, *supra* note 8, at §§ 74.05, 74.06.

<sup>11</sup> Of course, Cohen's "inductive" probabilities would allow an analogous interpretation of the relationship between the second sentence of the instruction and the first. See L. COHEN, *supra* note 1, at § 71.

<sup>12</sup> See, e.g., Allen, *supra* note 3, at 405-06.

<sup>13</sup> To date, defenses of the plausibility of a mathematicist interpretation have been developed primarily in the context of the gatecrasher paradox. See Kaye, *The Paradox of the Gatecrasher and Other Stories*, 1979 ARIZ. ST. L.J. 101; Cohen, *Subjective Probability and the Paradox of the Gatecrasher*, 1981 ARIZ. ST. L.J. 627;

The results of my informal survey of jury instructions shows that, on their face, most instructions are at least as ambiguous as the one discussed above. For example, reflect for a moment upon the following instruction for a simple negligence case:

In order to prove the essential elements of plaintiff's claim, the burden is on the plaintiff to establish, by a preponderance of the evidence in the case, the following facts: *First*, that the defendant was negligent in one or more of the particulars alleged; and *second*, that the defendant's negligence was a proximate cause of some injury and consequent damage sustained by the plaintiff.<sup>14</sup>

The ambiguities here are legion and obvious, though this instruction is somewhat more amenable to a mathematicist interpretation. Still, it is likely that lawyers do not appreciate these ambiguities. One should, therefore, consider such instructions skeptically, at least when considering which underlying theory of probability they confirm. While it is sometimes appropriate to assume that legal pronouncements carry subtle, even undiscovered, wisdom—or, as Cohen would have it, the wisdom of much everyday probabilistic reasoning—it is also sometimes true that those pronouncements simply lack logical sophistication.<sup>15</sup>

This is not to say that judges, or for that matter lay jurors or lawyers, have intuitions regarding the probabilistic evaluation of evidence that are largely erroneous. It is not at all uncommon to have reasonably correct intuitions but to be unable to explain them. It would not surprise me if the typical trier of fact, whether judge or jury, faced with the kind of situation described earlier (e.g.,  $\Pr(A) = .6$ ,  $\Pr(B) = .6$ , A and B independent) would find for the defendant, especially if any applicable jury instructions authorize such a result or are at all ambiguous on the matter.<sup>16</sup> On the other hand, it would be

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Kaye, *supra* note 2; see also Lempert, *The New Evidence Scholarship: Analyzing the Process of Proof*, 66 B.U.L. REV. 439 (1986).

<sup>14</sup> E. DEVITT & C. BLACKMAR, *supra* note 8, at § 80.17. See also *id.* at §§ 80.23, 82.02, 83.02, 84.03, 84.15, 90.21, 90.22, 90.26, 90.27, 90.34, 92.05.

<sup>15</sup> To corroborate this point, I want to report one of my experiences in going from an undergraduate education heavy in abstract mathematics to the rhetorical exercise of law school. I quickly found that lawyers had a propensity to infer the converse of an authoritative proposition, or not to infer it, in what seemed to be a random manner. A little later I realized that it was not random at all: the use of the inference was highly correlated with whether it got the speaker where she wanted to go. Of course, the logical error of inferring a proposition from (only) its converse is closely related to the confusion of necessity conditions and sufficiency conditions.

<sup>16</sup> Nor would it be surprising, however, if the case were found for the plaintiff, since ordinary people exercising intuitive judgment tend to overestimate the mathematical probability of the occurrence of conjunctive events, at least where the component events are probabilistically independent. See Saks & Kidd, *Human Information Processing and Adjudication: Trial by Heuristics*, 15 LAW & SOC'Y REV. 123, 142 (1980-81).

surprising to me if the trier were able to articulate its decision by reference to a rule as precise as Criterion I and to differentiate that from Criterion II. Such decisions by the trier of fact would help to render unnecessary the clarification of ambiguous instructions. Moreover, if such a decision were to arise in the context of a general verdict accompanied by interrogatories, the judge might well be obligated to assume a mathematicist interpretation in order to make the jury's answers to the interrogatories consistent with their general verdict.<sup>17</sup>

Of course, nothing about the argument presented here presupposes that mathematical probability is applicable to the logic of trials. That is an interesting and important issue which deserves all the discussion it is getting. Nevertheless, my tentative conclusion is that the relevant rules of law are ambiguous and thus do not supply much support for either position. Perhaps further research will require modification or abandonment of this conclusion. But if it is correct, then the debate should be pursued on grounds other than the relative compatibility of the contending theories with the established rules of law. Instead, each theory might better be used in the prescriptive enterprises of law reform and interpretation.

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<sup>17</sup> See J. FRIEDENTHAL, M. KANE & A. MILLER, CIVIL PROCEDURE 533 (1985).