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Innumeracy and Jurisprudence: The Surprising Difficulty of Counting Petition Signatures

Jonathan L. Entin

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Legal commentators often distinguish between bipolar and polycentric disputes, suggesting that the former are appropriate for judicial resolution whereas the latter are not.[1] Whatever its heuristic value, this dichotomy obscures the difficulty of drawing distinctions between archetypically simple and complex disputes. Some critics of the distinction have focused upon the similarities between recent forms of institutional litigation, which are often viewed as polycentric, and more traditional types of cases that have a distinguished legal pedigree.[2] This article approaches the question from the other side, suggesting that some problems that appear simple in fact have proven quite difficult for courts to address.

The difficulty is exemplified by the seemingly routine task of counting the number of valid signatures on petitions. Many states permit at least preliminary

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[1]Perhaps the most influential proponent of the view that polycentric disputes are inappropriate candidates for court disposition was Lon Fuller. *See* Lon L. Fuller, *The Forms and Limits of Adjudication*, 92 HARV. L. REV. 353, 394–404 (1978); *see also* DONALD L. HOROWITZ, THE COURTS AND SOCIAL POLICY (1977). For a recent suggestion that, regardless of the propriety of judicial involvement, courts generally cannot induce large-scale social change, see GERALD N. ROSENBERG, THE HOLLOW HOPE (1991).

[2]*E.g.*, Theodore Eisenberg & Stephen C. Yeazell, *The Ordinary and the Extraordinary in Institutional Litigation*, 93 HARV. L. REV. 465 (1980).

counts on the basis of verifications of a sample of all submitted signatures. Legislatures and courts have had difficulty in writing and interpreting sampling statutes, and electoral officials have had difficulty in implementing such laws.

Beyond its mathematical aspects, this subject raises larger questions about the respective roles of legislatures, agencies, and courts in the formulation and implementation of public policy. Moreover, it does so in the apparently mundane context of counting.[3] For that reason, it might help us to understand more clearly the issues of institutional competence and interaction that have been considered in more complex contexts. The surprising difficulty of counting valid signatures serves as a basis for reassessing the utility of the bipolar-polycentric dichotomy. Most "bipolar" legal disputes affect the interests and behavior of nonparties, and many "polycentric" disputes present issues that are traditionally seen as susceptible of judicial disposition. In short, attaching labels to disputes cannot substitute for careful analysis of available resolution devices.

I. BACKGROUND

Every state requires petitions signed by some minimum number of qualified voters to trigger certain aspects of the electoral process. Although the precise rules vary, they apply to nominations, recalls, initiatives, and referenda.[4] How can we tell whether a petition contains the requisite number of valid signatures?

At first blush, the answer seems obvious: check every signature against a master list of qualified voters. Unfortunately, life is not that simple. First, tens of thousands of signatures might be submitted. Moreover, no single master list may be available because voter registration records are usually maintained at the local level. When many signatures are submitted from several localities, the process of verification can become enormously complex. Accordingly, some states have statutes that allow officials to estimate the number of valid signatures by checking a sample of the submitted signatures.[5]

[3]Counting is often denigrated as trivial, but it is one of the most important things that governments (and scholars) do. *See* OTIS DUDLEY DUNCAN, NOTES ON SOCIAL MEASUREMENT 41–55, 150–51 (1984); *see also* IAN I. MITROFF ET AL., THE 1980 CENSUS: POLICYMAKING AMID TURBULENCE 2–4 (1983).

[4]*See generally* THOMAS E. CRONIN, DIRECT DEMOCRACY 2–3, 51, 62–66, 125–28 (1989).

[5]Sampling statutes fall into two general types. Most provide for a preliminary estimate of the number of valid signatures. Under this kind of statute, if the sample-based estimate suggests that the petition has obtained some fixed percentage above the required number of valid signatures, the measure or candidate qualifies for the ballot. If the estimate suggests a shortfall of a specified magnitude, the petition is disqualified. If the estimate falls between the thresholds for qualification or disqualification, every signature is checked. ARIZ. REV. STAT. ANN. §§ 11-135, 19-121.02, 19-121.04 (1990); CAL. ELEC. CODE §§ 3520-21, 3708, 6555.5, 6831.1, 27215 (West 1977, 1989 & Supp. 1993); CAL. PUB. UTIL. CODE § 12815.7 (West Supp. 1993); MO. ANN. STAT. §§ 116.120, 116.130 (Vernon Supp. 1992); NEV. REV. STAT. §§ 293.1277-.1279, 295.210 (1987); N.D. CENT. CODE § 16.1-01-10 (1991); WASH. REV. CODE ANN. §§ 29.79.200,

Other states apparently use sampling even without specific statutory authorization.[6]

The mathematical principles underlying this procedure are well established. We can reliably estimate how many members of a population have a particular characteristic by focusing upon an appropriately selected subset of the population. The basic subset for this purpose is known as a simple random sample.[7] In such a sample, every signature has an equal chance of being selected, and every possible combination of signatures has an equal chance of being chosen.[8] If those conditions are satisfied, the accuracy of the estimate can be calculated with some precision.[9] That is to say, we first decide how

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29.82.090(3) (Supp. 1993); *see also* N.D. CENT. CODE § 44-08-21 (Supp. 1991). Preliminary assessments based upon a sampling of submitted signatures generally are subject to judicial review.

Several states provide that the results of the sample analysis can determine whether a candidate, initiative, or referendum qualifies for the ballot without the necessity of a signature-by-signature check. FLA. STAT. ANN. § 99.097 (West 1982 & Supp. 1993); ILL. COMP. STAT. ANN. ch. 10, §§ 5/10-10, 28-10 to -13 (Smith-Hurd 1993); OR. REV. STAT. §§ 249.008(2), 250.105(4)-(5) (1991); TEX. ELEC. CODE ANN. §§ 141.069, 277.003 (Vernon Supp. 1993); *see also* D.C. CODE ANN. §§ 1-1312(*o*)(2a), 1-1320(*o*) (1987); ILL. ANN. STAT. ch. 46, ¶ 10-10 historical note (Smith-Hurd Supp. 1990). One of these statutes requires that the sampling be conducted such that the estimated number of valid signatures is accurate within a 99.5% level of statistical confidence. FLA. STAT. ANN. § 99.097(1)(b) (West 1982 & Supp. 1993). Another prohibits the disqualification of a petition for an insufficient number of valid signatures unless two different sample analyses, the second based upon a larger number of signatures than the first, support that conclusion. OR. REV. STAT. § 250.105(4) (1991). Some state election officials have promulgated regulations implementing these statutes. FLA. ADMIN. CODE § 1S-2.008 (1989); OR. ADMIN. R. 165-14-030 (1986). In one jurisdiction, no formal regulations have been promulgated, but officials use a computerized random-number generator to select the sample. Telephone conversation with Donald Schultz, Illinois Board of Elections, Aug. 6, 1990.

[6]*See, e.g.*, S.C. Att'y Gen. Op., Sept. 22, 1978 (discussing S.C. CODE ANN. § 7-11-70 (1977) (amended 1984)); *see also In re* Cook, 462 N.E.2d 557, 561 (Ill. App. Ct. 1984) (county board of elections uses random sample to estimate number of valid signatures on local initiative petition despite absence of express statutory authority for this practice). Some courts have considered the results of sample analyses in resolving challenges to be the number of valid signatures on petitions despite the absence of statutory authority for sampling. Four Thousand, Five Hundred Sixty-Eight Registered Voters v. City Clerk, 465 N.E.2d 1209, 1210 (Mass. 1984); Codd v. Barbaro, 443 N.Y.S.2d 683, 690 (Sup. Ct. 1981); *see also* CRONIN, *supra* note 4, at 237, 245 (urging that election officials use sampling techniques to verify signatures on initiative, referendum, and recall petitions).

[7]It is not essential to have a *simple* random sample to obtain statistically accurate information. Other kinds of samples are commonly used in academic, governmental, and commercial research. None of these alternative sampling techniques involves purely haphazard selection, however. Each uses randomness in the sense discussed in the text to generate data of statistically measurable accuracy. *See* HUBERT M. BLALOCK, JR., SOCIAL STATISTICS 558, 560, 567 (rev. 2d ed. 1979); LESLIE KISH, SURVEY SAMPLING 75, 113, 148 (1965); C.A. MOSER & G.F. KALTON, SURVEY METHODS IN SOCIAL INVESTIGATION 85, 101 (2d ed. 1972).

In fact, most of the sampling statutes considered here authorize verification of samples selected at the county or other subdivision level. Variations among kinds of random samples are not significant for the present discussion.

[8]*See* BLALOCK, *supra* note 7, at 140; KISH, *supra* note 7, at 36–40; MOSER & KALTON, *supra* note 7, at 63, 80–81.

[9]*See generally* BLALOCK, *supra* note 7, at 140, 183–86. Perhaps the most notorious example of a nonrandom sample generating inaccurate data is the *Literary Digest* survey that predicted a Landon landslide in the 1936 presidential election. *See* MOSER & KALTON, *supra* note 7, at 79; JOHN ALLEN PAULOS, INNUMERACY 151 (1988); JEFFREY A. WITMER, DATA ANALYSIS 97 (1992).

many signatures to examine and then select them in a way that gives every signature on every petition form the same probability of being examined. Otherwise, we cannot have confidence that the proportion of valid signatures in the sample reflects the proportion of valid signatures in the total pool.[10]

[10]Having a random sample does not, by itself, tell us how much confidence we should have in the estimate it generates. The accuracy of an estimate also depends upon the size of the sample. This is so because samples almost never mirror the population from which they are drawn. Instead, samples approximate the population but almost always differ from it to some extent. Therefore, we should expect any estimate of the number of valid petition signatures based upon a random sample of the total pool of signatures to be at least somewhat inaccurate no matter how carefully the sample was drawn.

If, however, we were to examine many random samples drawn from the entire pool of signatures on a petition, we would find that the proportion of valid signatures in the samples tends to cluster around the actual proportion of valid signatures on the petition. Moreover, the distribution of sample figures would be approximately normal. BLALOCK, *supra* note 7, at 183. A normal distribution is bell-shaped and symmetrical, with the frequency of sample observations dispersed in a known mathematical pattern around the actual proportion of valid signatures. *Id.* at 92–95; MOSER & KALTON, *supra* note 7, at 72–73.

The dispersion of a normal distribution is measured in units of standard deviation, or in the case of samples, standard error. BLALOCK, *supra* note 7, at 180; KISH, *supra* note 7, at 11. The pattern of dispersion in any normal distribution is such that slightly more than two-thirds of all cases fall within one unit of standard error of the actual proportion of valid signatures in the total pool, slightly more than 95% of all cases fall within two standard errors, and 99.7% of all cases fall within three standard errors. BLALOCK, *supra* note 7, at 95–96; MOSER & KALTON, *supra* note 7, at 72; R.J. SENTER, ANALYSIS OF DATA 92–94 (1969). The standard error varies inversely with the square root of the size of the sample. In other words, the larger the sample size, the smaller the standard error. *See* BLALOCK, *supra* note 7, at 180, 182–86.

Of course, we would not examine many random samples drawn from the entire pool of signatures on a petition, because that would neutralize the value of sampling. We also do not know the actual number of valid signatures, which is what we are trying to estimate from the sample. Fortunately, the normality of the sampling distribution—the distribution of the relative number of times we would expect to obtain a particular proportion of valid signatures in a large number of hypothetical random samples—enables us to assess how well the sample figure approximates the actual figure. *See id.* at 153; KISH, *supra* note 7, at 10; MOSER & KALTON, *supra* note 7, at 64; SENTER, *supra*, at 124–25.

The larger the sample, the more confident we can be that the sample figure represents an accurate estimate of the actual figure. How large should the sample be? Statisticians have no fixed answer to this question. It depends upon how we balance our concerns for accuracy and for efficiency, because estimates using larger samples tend to be more accurate but also to take longer and cost more to complete. BLALOCK, *supra* note 7, at 215–18; KISH, *supra* note 7, at 24–25; MOSER & KALTON, *supra* note 7, at 146–52. Inaccurate decisions come in two varieties. First, the sample-based estimate might mistakenly suggest that a petition does not contain enough valid signatures, so a qualified candidate or proposition would not be placed on the ballot. An incorrect rejection is called a type I error. Second, the sample-based estimate might mistakenly suggest that a petition does contain enough valid signatures, so an unqualified candidate or proposition would be placed on the ballot. An incorrect acceptance is called a type II error. BLALOCK, *supra* note 7, at 109–10; SENTER, *supra*, at 185. (There is actually a third source of inaccuracy. Errors could arise in the count of valid signatures. For example, some signatures that are in fact valid might be rejected, and other signatures that really are invalid might be accepted. These are measurement rather than sampling errors; they can arise whether a subset or all of the signatures are checked. For that reason, this source of inaccuracy does not affect decisions about sample size.) Unfortunately, for a fixed sample size we cannot simultaneously minimize the risk of both sorts of error. BLALOCK, *supra* note 7, at 159. Accordingly, we must decide whether to err in the direction of excessively permissive or excessively restrictive ballot access. Resolving these issues is beyond the scope of this article.

Constructing a random sample is relatively straightforward. Each signature can be assigned an identifying number, and sample signatures then can be selected through the use of standard tables of random numbers.[11] It appears, however, that election officials do not always select their samples according to the statistical criteria for randomness. For example, several states use computer software that designates specific lines on the printed petition forms for inclusion in the sample, but not every line contains a signature. These blank lines should be entirely disregarded.[12] At least two states, however, select the next line that contains a signature.[13] This procedure gives those lines an increased probability for inclusion in the sample, thereby violating the equal-likelihood criterion for a random sample. It is unclear to what extent this practice introduces statistical bias, although there are circumstances in which unequal probability of inclusion is known to produce biased samples.[14]

We should not attach much legal significance to this departure from statistical norms. Courts have generally rejected challenges to governmental decisions based upon more serious violations of randomness criteria. Perhaps the most notable example is the first draft lottery held during the Vietnam War, which was conducted in a way that produced a nonrandom order of priority for induction into the armed forces.[15] Nevertheless, the lottery was upheld on the theory that the procedures were adequate even if scientifically imperfect.[16] Analogous decisions have resulted in other areas.[17] Still, this phenomenon

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[11]BLALOCK, *supra* note 7, at 556; KISH, *supra* note 7, at 27; MOSER & KALTON, *supra* note 7, at 82, 152–54. Alternatively, the sample can be constructed by selecting the first signature at random and choosing subsequent signatures at a fixed interval thereafter (e.g., every tenth name). This system will satisfy the statistical criteria for randomness only if the pool of signatures is itself in random order. MOSER & KALTON, *supra* note 7, at 81.

[12]KISH, *supra* note 7, at 37.

[13]Arizona Secretary of State, Initiative/Referendum Petition Procedures, June 19, 1990, at 3; OR. ADMIN. R. 165-14-030(5) (1986).

[14]BLALOCK, *supra* note 7, at 555; MOSER & KALTON, *supra* note 7, at 82.

[15]Stephen E. Fienberg, *Randomization and Social Affairs: The 1970 Draft Lottery*, 171 SCIENCE 255, 257–60 (1971).

[16]United States v. Kotrlik, 465 F.2d 976, 977–78 (9th Cir.) (per curiam), *cert. denied*, 409 U.S. 1043 (1972); *see also* United States v. Johnson, 473 F.2d 677, 678 (9th Cir. 1972) (per curiam); United States v. Battin, 466 F.2d 1194, 1194 (9th Cir. 1972) (per curiam).

[17]*See, e.g.*, United States v. Eyster, 948 F.2d 1196, 1213 (11th Cir. 1991) (upholding creation of jury venire from persons with surnames beginning with only certain letters of alphabet against claim that selection was not random); United States v. Proceeds of the Sale of 9,312 Pounds of Scallops, 738 F. Supp. 598, 601–03 (D. Mass. 1990) (upholding forfeiture against challenge that inspection had failed to examine truly random sample of catch). *But see* Waddell v. State, 654 S.W.2d 752, 753 (Tex. Ct. App. 1983) (finding examination of sacked oysters but not unsacked oysters violated requirement for inspection of random sample of entire catch).

These are not the only examples in which government agencies have had difficulty implementing basic sampling principles. *See* William T. Bogart, *Economic Implications of Tax Administration: Property Assessment, Equalization, and School Aid in New Jersey*, 10 PROP. TAX. J. 377, 380–89 (1991).

Despite these difficulties, one court has begun to experiment with sampling in the mass-tort context. Cimino v. Raymark Indus., 751 F. Supp. 649 (E.D. Tex. 1990); *see* Michael J. Saks & Peter David Blanck, *Justice Improved: The Unrecognized Benefits of Aggregation and Sampling in the Trial of Mass Torts*, 44 STAN. L. REV. 815, 841–51 (1992).

implies that the task of counting valid signatures might not be so simple after all. A more troublesome aspect of the task has arisen, however, and it is to that problem that we now turn.

II. EXTRAPOLATING FROM SAMPLE TO UNIVERSE

The Supreme Court of Arizona recently had occasion to construe one of these sampling statutes. The result was not a happy one. Both the court and the parties made basic mathematical errors. Some of those errors arose from infelicitous legislative drafting, but the difficulties reflect a more pervasive lack of quantitative sophistication among bench and bar.

Under Arizona law, whenever an initiative or referendum petition is submitted to the secretary of state, that official must randomly select five percent of the signatures for verification.[18] If the projected number of valid signatures is at least five percent above the minimum required by law, the petition is placed on the ballot.[19] If the projected number of valid signatures is at least five percent below the legal minimum, the petition is returned to its sponsors.[20] If the projected number of valid signatures is within five percent of the legal minimum, every signature is checked.[21]

Estimating the number of valid signatures is complicated by two features of the statutory scheme. First, a signature obtained by an ineligible petition circulator does not count, even if the signer is a qualified voter and therefore eligible to sign.[22] Second, the statutory formula for calculating the estimated number of valid signatures based upon the results of the sampling was poorly drafted. We shall return to this point after a necessary digression.

A. Preliminary Considerations

Let us first consider how to determine the number of valid signatures in the absence of the sampling statute. We would examine each name, eliminating anyone whose signature was obtained by an ineligible circulator or who is not a qualified voter. In symbolic terms,

$$V = T - (C + S) \qquad [1]$$

where

V = the number of valid signatures;
T = the total number of signatures submitted;
C = the number of signatures obtained by ineligible circulators; and
S = the number of signatures by ineligible signers.

[18] ARIZ. REV. STAT. ANN. § 19-121.01(B) (Supp. 1992).
[19] *Id.* § 19-121.04(B).
[20] *Id.* § 19-121.04(D).
[21] *Id.* § 19-121.04(C). If the signature-by-signature check cannot be completed before the deadline for printing ballots, the petition will be presumed to contain the requisite number of signatures. Save Our Public Lands Coalition v. Stover, 662 P.2d 136, 139 (Ariz. 1983).
[22] ARIZ. REV. STAT. ANN. § 19-114 (1990 & Supp. 1992).

This point can be illustrated through a hypothetical example, which will serve as the basis for much of the following discussion. Assume that: (1) a petition containing 20,000 signatures is submitted (*T*); (2) 4,000 of those signatures were obtained by ineligible circulators (*C*); and (3) 6,000 signatures came from ineligible signers (*S*). Focusing only upon the invalid signatures for the moment, we see that

$$C + S = 4{,}000 + 6{,}000 = 10{,}000.$$

To determine the number of valid signatures, we find that

$$\begin{aligned} V &= T - (C + S) \\ &= 20{,}000 - (4{,}000 + 6{,}000) \\ &= 20{,}000 - 10{,}000 \\ &= 10{,}000. \end{aligned}$$

The preceding discussion assumes that there is no overlap between the classes of signatures that are invalid because they were obtained by ineligible circulators (*C*) and because they came from ineligible signers (*S*). However, these classes are not necessarily mutually exclusive. Some signers might be ineligible in both ways: their signatures were obtained by ineligible circulators (so these persons are included in *C*), and they are not qualified voters (so these persons also are ineligible signers included in *S*). In order to prevent double deduction of invalid signatures, we must then adjust our calculation to take account of those signatures that are invalid on both grounds. We can do that as follows:

$$V = T - (C + S - B) \qquad [2]$$

where

V = the number of valid signatures;
T = the total number of signatures submitted;
C = the number of signatures obtained by ineligible circulators;
S = the number of signatures by ineligible signers; and
B = the number of signatures that are invalid on both grounds (i.e., obtained by ineligible circulators from ineligible signers).

This point can be illustrated through the addition of another element to our hypothetical example. Assume that: (1) a petition containing 20,000 signatures is submitted (*T*); (2) 4,000 of those signatures were obtained by ineligible circulators (*C*); (3) 6,000 of those signatures came from ineligible signers (*S*); and (4) 1,000 of the ineligible signatures were obtained by ineligible circulators (*B*). Again, focusing only upon the invalid signatures, we see that

$$C + S = 4{,}000 + 6{,}000 = 10{,}000.$$

Notice, however, that 1,000 of the 4,000 persons included in C also are included in S, and that 1,000 of the 6,000 persons included in S also are included in C. The correct number of invalid signatures therefore is really

$$I = C + S - B \qquad [3]$$

where

I = the total number of invalid signatures;
C = the number of signatures obtained by ineligible circulators;
S = the number of signatures by ineligible signers; and

B = the number of signatures that are invalid on both grounds (i.e., obtained by ineligible circulators from ineligible signers).

Applying equation 3, we find that

$$C + S - B = 4{,}000 + 6{,}000 - 1{,}000 = 9{,}000.$$

Alternatively, we could say that some signatures are invalid solely because they were obtained by ineligible circulators, others are invalid solely because they came from ineligible signers, and the remainder are invalid on both grounds. We can represent this situation symbolically as

$$I = (C - B) + (S - B) + B \quad [4]$$

where

I = the total number of invalid signatures;

$(C - B)$ = the number of signatures obtained by ineligible circulators from eligible signers;

$(S - B)$ = the number of signatures by ineligible signers obtained by eligible circulators; and

B = the number of signatures that are invalid on both grounds (i.e., obtained by ineligible circulators from ineligible signers).

A moment's reflection will show that equations 3 and 4 are algebraically equivalent. We can rewrite equation 4 as follows:

$$\begin{aligned} I &= (C - B) + (S - B) + B \\ &= C - B + S - B + B \\ &= C + S - 2B + B \\ &= C + S - B. \end{aligned}$$

In this example, 3,000 signatures are invalid solely because they were obtained by ineligible circulators, and another 5,000 signatures are invalid solely because the signers were not qualified voters, while 1,000 signatures are invalid on both grounds. Applying equation 4, we see that

$$\begin{aligned} I &= (C - B) + (S - B) + B \\ &= (4{,}000 - 1{,}000) + (6{,}000 - 1{,}000) + 1{,}000 \\ &= 3{,}000 + 5{,}000 + 1{,}000 \\ &= 9{,}000. \end{aligned}$$

Hence, equations 3 and 4 yield the same result.

To determine the number of valid signatures, we can use equation 2 to show that

$$\begin{aligned} V &= T - (C + S - B) \\ &= 20{,}000 - (4{,}000 + 6{,}000 - 1{,}000) \\ &= 20{,}000 - 9{,}000 \\ &= 11{,}000. \end{aligned}$$

B. Alternative Interpretations of the Arizona Sampling Statute

The verbal formula given in the Arizona sampling statute for estimating the number of valid signatures tells us to

> subtract from the number of signatures [submitted] . . . all signatures included in the random sample . . . found to be ineligible, and . . . , after determining the percentage of signatures found to be invalid in the random sample, subtract a like percentage of all other signatures included on the petitions and all signatures appearing upon signature sheets circulated by persons who . . . were not [eligible to circulate the petition].[23]

In *City of Flagstaff v. Mangum*,[24] three different interpretations of this formula were advanced. Equations 5, 8, and 10 embody those interpretations. Although these equations are the author's, they were derived from the court's opinion.[25] The following discussion and numerical examples, which are also the author's, show that all of the interpretations are mathematically unsound.

1. *The City's Approach*

The city argued for the following approach:

$$V_e = T - s - (s/t)(T - s) - C \qquad [5]$$

where

V_e = the estimated number of valid signatures;
T = the total number of signatures submitted;
s = the number of signatures in the sample by ineligible signers;
t = the total number of signatures in the sample; and
C = the number of signatures obtained by ineligible circulators.

The city's approach purports to follow the literal language of the statute. Equation 5 begins with the "number of signatures [submitted]" (T), subtracts "all signatures included in the random sample . . . found to be ineligible" (s), determines "the percentage of signatures found to be invalid in the random sample" (s/t), subtracts "a like percentage" from "all other signatures included on the petitions" ($T - s$), and finally subtracts "all signatures appearing upon signature sheets circulated by persons who . . . were not [eligible to circulate the petition]" (C).

To understand the city's approach, let us consider our hypothetical example—20,000 signatures (T), 4,000 of which were obtained by ineligible circulators (C), 6,000 of which came from ineligible signers (S), and 1,000 of which were invalid on both grounds (B)—just a bit further. Assume that: (1) a five percent random sample of the total of 20,000 signatures is checked; (2) of these 1,000 signatures (t), 200 were obtained by ineligible circulators (c); (3) 300 of those signatures came from ineligible signers (s); and (4) 50 of the ineligible

[23]ARIZ. REV. STAT. ANN. § 19-121.04(A) (1990). While this article was in press, that provision was rewritten as part of a comprehensive revision of Arizona's election laws. Although the amended formula is a significant improvement over the original version, *see infra* note 51, the statutory change does not affect this article's jurisprudential analysis.

[24]793 P.2d 548 (Ariz. 1990).

[25]The court's explanation of the alternative interpretations of this statutory language appears almost entirely in tabular form. The tables contain calculations of the estimated number of valid signatures in the petition at issue based upon the alternative interpretations of the statute. *See id.* at 555, 557.

signatures were obtained by ineligible circulators (b). Using equation 5, the city would estimate the number of valid signatures as follows:

$$
\begin{aligned}
V_e &= T - s - (s/t)(T - s) - C \\
&= 20{,}000 - 300 - (300/1{,}000)(20{,}000 - 300) - 4{,}000 \\
&= 20{,}000 - 300 - (.3)(19{,}700) - 4{,}000 \\
&= 20{,}000 - 300 - 5{,}910 - 4{,}000 \\
&= 9{,}790.
\end{aligned}
$$

Because we know from our earlier discussion that the actual number of valid signatures in this hypothetical example is 11,000, we can conclude that the city's estimate is inaccurate. The inaccuracy of the estimate is not an artifact of a biased sample. The sample figures precisely reflect the actual figures; each of the sample numbers is exactly five percent of the corresponding actual number.[26] Thus, the problem arises from intrinsic deficiencies in the city's approach.

The problem with the city's approach is that equation 5 improperly subtracts some signatures twice. Most important, this approach fails to recognize that some signatures might be invalid both because they were obtained by ineligible circulators and because the signers were ineligible. The discussion of equation 2 explained why the failure to take account of these "double ineligibles" results in an overstatement of the number of invalid signatures. By ignoring the overlap between the two classes of invalid signatures, equation 5 necessarily produces an inaccurate estimate of the number of valid signatures.

The difficulties do not end there. Another distortion results from subtracting the number of signatures obtained by ineligible circulators (C) as the last step in the calculation. To understand why this is problematical, we need to examine the city's approach in more detail. According to equation 5,

$$V_e = T - s - (s/t)(T - s) - C.$$

Recall, however, that equation 2 tells us that we can calculate the number of valid signatures in the absence of a sampling statute as

$$V = T - (C + S - B).$$

This means that we must subtract the number of signatures obtained by ineligible circulators (C) from the total number of signatures (T), but it means that we should do so only once. Solving equation 2 for T, we find that

$$T = V + C + S - B. \qquad [6]$$

The value of T established in equation 6 can be substituted in equation 5 to show that equation 5 oversubtracts signatures obtained by ineligible circulators (C):

$$
\begin{aligned}
V_e &= T - s - (s/t)(T - s) - C \\
&= (V + C + S - B) - s - (s/t)[(V + C + S - B) - s] - C \\
&= V + S - B - s - (s/t)[(V + C + S - B) - s] + C - C \\
&= V + S - B - s - (s/t)[(V + C + S - B) - s]. \qquad [7]
\end{aligned}
$$

[26]It is, of course, exceedingly unlikely that a random sample will mirror the actual pool of signatures so precisely. *See supra* note 10. The sample numbers in the text were chosen for ease of exposition only.

To put the point in verbal terms, equation 7 is the algebraic equivalent of equation 5. The number of signatures obtained by ineligible circulators (C), which is included in the total number of signatures (T), must be subtracted only once if we are to estimate accurately the number of valid signatures. A single subtraction would cause C to drop out of the picture entirely. This does not happen under the city's approach, as the expression $[(V + C + S - B) - s]$ in equation 7 shows. This expression means that some fraction (s/t) of the signatures obtained by ineligible circulators (C) is subtracted a second time.[27] It is possible to adjust equation 5 to correct this problem, a point to which we shall return.

The problem with equation 5 is intrinsic, and it is not trivial. If the double deduction necessarily overstates the number of invalid signatures, it simultaneously understates the number of valid signatures. In some circumstances, the city's approach can produce truly absurd results. For example, suppose that we have a second hypothetical petition containing 20,000 signatures (T). Unlike our paradigm hypothetical, this one contains 4,000 signatures obtained by ineligible circulators (C) and 16,000 signatures from ineligible signers (S); 2,000 signatures are invalid on both grounds (B).[28] Assume once more that the five percent random sample of signatures drawn for verification reflects the actual situation perfectly: of the 1,000 sampled signatures (t), 200 were obtained by ineligible circulators (c), 800 come from ineligible signers (s), and 100 are invalid on both grounds (b). Applying equation 5, we estimate the number of valid signatures as follows:

$$
\begin{aligned}
V_e &= T - s - (s/t)(T - s) - C \\
&= 20{,}000 - 800 - (800/1{,}000)(20{,}000 - 800) - 4{,}000 \\
&= 20{,}000 - 800 - (.8)(19{,}200) - 4{,}000 \\
&= 20{,}000 - 800 - 15{,}360 - 4{,}000 \\
&= -160.
\end{aligned}
$$

This result speaks for itself. It is logically impossible for the number of valid signatures to be less than zero, yet equation 5 gives a negative estimate in this case.[29]

[27]The failure of equation 5 to consider the problem of the "double ineligibles" discussed above also means that the fraction s/t inaccurately estimates the percentage of invalid signatures in the sample. The impact of this mistake will vary depending upon the extent of the overlap between those signatures that are invalid because they were obtained by ineligible circulators and those that are invalid because they came from ineligible signers.

[28]Applying equation 2, we can calculate the actual number of valid signatures as follows:

$$
\begin{aligned}
V &= T - (C + S - B) \\
&= 20{,}000 - (4{,}000 + 16{,}000 - 2{,}000) \\
&= 20{,}000 - 18{,}000 \\
&= 2{,}000.
\end{aligned}
$$

[29]As a general proposition, equation 5 always will produce a negative estimate of the number of valid signatures whenever

$$(ST/t) + C \geq T$$

where

s = the number of signatures in the sample by ineligible signers;
T = the total number of signatures submitted;
t = the total number of signatures in the sample; and
C = the number of signatures obtained by ineligible circulators

Therefore, the city's approach must be rejected.[30]

2. *The Sponsors' Approach*

Equation 5 contains another flaw that will be easier to see by focusing upon the interpretation advanced by the sponsors of the petition. They proposed the following approach:

$$V_e = T - C - (s - b) - [(s - b)/(t - c)]\ [T - C - (s - b)] \quad [8]$$

where

V_e = the estimated number of valid signatures;
T = the total number of signatures submitted;
C = the number of signatures obtained by ineligible circulators;
s = the number of signatures in the sample by ineligible signers;
b = the number of signatures in the sample obtained by ineligible circulators from ineligible signers;
t = the total number of signatures in the sample; and
c = the number of signatures in the sample obtained by ineligible circulators.

The sponsors' approach does not track the statutory language as closely as does the city's, although it departs from the literal words of the statute less drastically than a first reading might suggest. The sponsors begin with "the number of signatures [submitted]" (T). After reordering the provision for subtracting "all signatures appearing upon signature sheets circulated by persons who . . . were not [eligible to circulate the petition]" (C), this approach returns to the statutory sequence to subtract "all signatures included in the random sample . . . found to be ineligible" ($s - b$), determine "the percentage of signatures found to be invalid in the random sample" $[(s - b)/(t - c)]$ and subtract "a like percentage" from "all other signatures included on the petitions" ($[T - C - (s - b)]$). Note that the sponsors define these latter three terms in somewhat different algebraic terms than did the city. These definitional differences reflect the greater mathematical sophistication embodied in the sponsors' approach.[31]

In fact, equation 8 differs from equation 5 in several important respects. One

when both s and C are greater than zero. If either s or C is zero, equation 5 will produce a negative estimate of the number of valid signatures only if

$$(ST/a) + C > T$$

For any petition containing 20,000 signatures that is subject to a five percent random sample, equation 5 will result in a negative estimate whenever

$$20s + C \geq 20{,}000$$

because

$$T/t = 20{,}000/1{,}000 = 20.$$

[30] "From a statistical perspective, if a model gives impossible estimates, then the model is wrong and its results are untrustworthy even when they fall into the range of the possible." Stephen P. Klein et al., *Ecological Regression* versus *the Secret Ballot*, 31 JURIMETRICS J. 393, 399 (1991) (footnote omitted).

[31] To say that the sponsors showed more mathematical sophistication than did the city does not mean that the sponsors avoided analytical error. *See infra* text accompanying note 32.

is that equation 8 eliminates all of the signatures obtained by ineligible circulators (C) at the beginning rather than at the end of the calculation. Correlatively, this approach addresses the difficulty we saw in equation 7 by eliminating signatures obtained by ineligible circulators (C) from the calculation. This can be seen algebraically if we use equation 6 to make the same substitution in equation 8 that was made in equation 5. Thus, equation 6 tells us that

$$T = V + C + S - B.$$

Substituting this value of T from equation 6 into equation 8, we find that

$$\begin{aligned} V_e &= T - C - (s - b) - [(s - b)/(t - c)][(T - C - (s - b)] \\ &= (V + C + S - B) - C - (s - b) \\ &\quad - [(s - b)/(t - c)][(V + C + S - B) - C - (s - b)] \\ &= V + C - C + S - B - (s - b) \\ &\quad - [(s - b)/(t - c)][V + C - C + S - B - (s - b)] \\ &= V + S - B - (s - b) \\ &\quad - [(s - b)/(t - c)][(V + S - B - (s - b)] \qquad [9] \end{aligned}$$

In other words, C has completely disappeared from equation 9.[32] Because equation 9 is algebraically equivalent to equation 8, this means that the sponsors' approach avoids the problem of double subtraction of signatures obtained by ineligible circulators (C) that we saw in the city's approach.

Another important difference between equations 5 and 8 is that equation 8 recognizes the overlap between the classes of signatures that are invalid because they were obtained by ineligible circulators (C in the entire pool of signatures, c in the sample) and those that are invalid because the signers were ineligible (s). This is shown by the inclusion of the variable b, the extent of the overlap in the sample, in equation 8. In short, this approach incorporates the lessons concerning the "double ineligibles" that we learned in connection with equation 2.

To understand the sponsors' approach and how it differs from the city's, let us return to our paradigm hypothetical example.[33] Applying equation 8, we can estimate the number of valid signatures as follows:

$$\begin{aligned} V_e &= T - C - (s - b) - [(s - b)/(t - c)][T - C - (s - b)] \\ &= 20{,}000 - 4{,}000 - (300 - 50) - \\ &\qquad [(300 - 50)/(1{,}000 - 200)]\,[20{,}000 - 4{,}000 - (300 - 50)] \\ &= 20{,}000 - 4{,}000 - 250 - [(250/800)]\,[20{,}000 - 4{,}000 - 250] \\ &= 20{,}000 - 4{,}000 - 250 - (.3125)(15{,}750) \\ &= 20{,}000 - 4{,}000 - 250 - 4{,}922 \\ &= 10{,}828. \end{aligned}$$

[32]While C has disappeared from equation 9, the number of signatures in the sample obtained by ineligible circulators (c) has not. This fact does not contradict the statement in the text. The presence of c in $[(s - b)/(t - c)]$ is necessary to calculate accurately the percentage of invalid signatures in the sample.

[33]Recall that this example involves: a petition containing 20,000 signatures (T); 4,000 of which were obtained by ineligible circulators (C); 6,000 of which came from ineligible signers (S); and 1,000 of which are invalid on both grounds (B). We also have: a five percent random sample, or 1,000 signatures (t); 200 of which were obtained by ineligible circulators (c); 300 of which came from ineligible signers (s); and 50 of which are ineligible on both grounds (b).

This estimate of the number of valid signatures is more accurate than the one obtained using equation 5, but it too differs from the actual number, which we know is 11,000. Once again, we can conclude that the inaccuracy of this estimate is not an artifact of a biased sample, because the sample numbers still perfectly reflect the actual numbers.

The difficulty with the sponsors' approach is that equation 8 incorrectly defines the base from which the final calculation is made. That base is defined as the total number of signatures submitted less all signatures obtained by ineligible circulators less signatures in the sample that are invalid solely because the signer was ineligible $[T - C - (s - b)]$. Careful attention to the language and underlying mathematical logic of the statute will demonstrate the error. The statute tells us that we should first subtract "all signatures in the random sample . . . found to be ineligible" and then, "after determining the percentage of signatures found to be invalid in the random sample, subtract a like percentage of *all other signatures included on the petitions*."[34] Because the percentage of invalid sample signatures is based upon all of the signatures in the sample, "all other signatures included on the petitions" must exclude all of the signatures in the sample. Equation 8 excludes only those signatures in the sample that are invalid solely because the signer was ineligible $(s - b)$. Hence, the number of excluded signatures is too small, and the projected number of invalid signatures (the "like percentage" multiplied by the base of "all other signatures") is too large. Consequently, equation 8 also must underestimate the number of valid signatures.

The same difficulty appears in equation 5; this is the other flaw in the city's approach mentioned earlier. We already have seen several other serious problems with that approach. Equation 5 defines "all other signatures" even less accurately than does equation 8 as the total number of signatures submitted less the number of invalid signatures in the sample regardless of the eligibility of the circulator $(T - s)$.

Equation 8 is much less severely flawed than is equation 5. Nevertheless, it does not accurately estimate the number of valid signatures on a petition. As was mentioned in connection with equation 5, it is possible to amend equation 8 so that it will produce correct projections. We shall return to this point.

3. *The Court's Approach*

The court rejected both of the approaches that have been discussed thus far in favor of its own alternative. The judicially approved interpretation of the statutory formula is:

$$V_e = T - C - s - (s/t)(T - C - s) \qquad [10]$$

where

V_e = the estimated number of valid signatures;
T = the total number of signatures submitted;
C = the number of signatures obtained by ineligible circulators;

[34] ARIZ. REV. STAT. ANN. § 121.04(A) (1990) (emphasis added).

s = the number of signatures in the sample by ineligible signers; and
t = the total number of signatures in the sample.

This approach represents something of a compromise between the alternatives proposed by the parties. In order to avoid the distortions in the city's approach created by subtracting the number of signatures obtained by ineligible circulators (C) as the last step in the calculation, the court follows the sponsors in subtracting C at the outset. At the same time, the court follows the city in calculating the "percentage of signatures found to be invalid in the random sample" (s/t). Finally, the court determines the base of "all other signatures included on the petitions" ($T - C - s$) in a way that differs from both the city's and the sponsors' approaches.

To understand the judicially sanctioned interpretation of the statutory formula, let us return one last time to our paradigm hypothetical example.[35] Using equation 10, we can estimate the number of valid signatures as follows:

$$\begin{aligned} V_e &= T - C - s - (s/t)(T - C - s) \\ &= 20{,}000 - 4{,}000 - 300 - (300/1{,}000)(20{,}000 - 4{,}000 - 300) \\ &= 20{,}000 - 4{,}000 - 300 - (.3)(15{,}700) \\ &= 20{,}000 - 4{,}000 - 300 - 4{,}710 \\ &= 10{,}990. \end{aligned}$$

This figure is very close to the actual number of valid signatures in our paradigm hypothetical, which we already have seen is 11,000. This fact suggests that the court's approach is preferable to the alternatives. Two caveats are in order. First, equation 10 does not in fact produce a completely correct estimate; this alone should suggest that this approach is not entirely trustworthy. Second, estimates produced by equation 10 under other hypothetical facts are much less accurate than the one we have just seen.

To grasp the shortcomings of the court's approach, let us return to the example in which equation 5 produced a negative estimate of the number of valid signatures.[36] Applying equation 10, we estimate the number of valid signatures as follows:

$$\begin{aligned} V_e &= T - C - s - (s/t)(T - C - s) \\ &= 20{,}000 - 4{,}000 - 800 - (800/1{,}000)(20{,}000 - 4{,}000 - 800) \\ &= 20{,}000 - 4{,}000 - 800 - (.8)(15{,}200) \\ &= 20{,}000 - 4{,}000 - 800 - 12{,}160 \\ &= 3{,}040. \end{aligned}$$

[35] Again, this example involves: a petition containing 20,000 signatures (T); 4,000 of which were obtained by ineligible circulators (C); 6,000 of which came from ineligible signers (S); and 1,000 of which are invalid on both grounds. We also have: a five percent random sample, or 1,000 signatures (t); 200 of which were obtained by ineligible circulators (c); 300 of which came from ineligible signers (s); and 50 of which are ineligible on both grounds (b).

[36] That example involves: a petition containing 20,000 signatures (T), 4,000 of which were obtained by ineligible circulators (C), 16,000 of which came from ineligible signers (S), and 2,000 of which are invalid on both grounds (B); the five percent random sample of 1,000 signatures (t) includes 200 that were obtained by ineligible circulators (c), 800 that came from ineligible signers (s), and 100 that are invalid on both grounds (b).

We know, however, that the actual number of valid signatures in this example is 2,000.[37] Equation 10, therefore, results in a very inaccurate estimate in this case, although the error is an overstatement rather than an understatement of the number of valid signatures.

As with equations 5 and 8, the inaccuracy of these estimates results from intrinsic deficiencies in the court's approach. The primary defect in equation 10 is that it ignores the possibility that some signatures might be invalid both because they were obtained by ineligible circulators and because they came from ineligible signers. We can see that the court paid no attention to the overlap because equation 10 omits any measure of this variable (B in the pool of all signatures, b in the sample).

The impact of this oversight cannot be assessed with generality. Ignoring the "double ineligible" problem infects the definition of both the "like percentage" of invalid sample signatures and "all other signatures included on the petitions" to which the percentage is applied. The precise effect will vary, of course. We have seen that equation 10 can either overestimate or underestimate the number of valid signatures. The court's approach, like the others, is clearly unsatisfactory. We therefore turn next to accurate methods for estimating the number of valid signatures.

C. Untangling the Arizona Statute

No single solution to these problem exists. Several mathematically equivalent alternatives would have allowed the parties and the court to avoid the traps that ensnared them in *City of Flagstaff v. Mangum*. The following discussion offers a straightforward method for estimating the number of valid petition signatures from a properly selected sample of the pool of signatures submitted, and explains how the Arizona sampling statute should have been interpreted to avoid the difficulties discussed in the preceding section.

Before turning to the proper interpretation of the Arizona sampling statute or the optimal way to adjust each of the equations embodying the alternative interpretations of that statute, we should understand the precise task at hand. Clear thinking in mathematics, as in other endeavors, often begins with a picture.[38] We therefore need a mental image of our inquiry.

To put the matter in the most general terms, there are only two kinds of signatures on a petition: valid and invalid. Signatures can be invalid for many reasons. As we have seen, some might come from persons who are not registered voters, and others might be obtained by persons who are not permitted to circulate petitions. In fact, the grounds for invalidity can be broader than these. For example, the signer might not use the full name appearing on the

[37] *See supra* note 28.

[38] IVAR EKELAND, MATHEMATICS AND THE UNEXPECTED 7-9 (1988).

voter registration list,[39] omit his address or voting district,[40] sign the petition more than once,[41] or simply write illegibly.[42] The circulator might obtain signatures on the same signature sheet from voters registered in different jurisdictions or electoral districts,[43] neglect to execute the required affidavit attesting to compliance with applicable legal requirements,[44] or fail to submit all signature sheets to election officials simultaneously.[45] Distinguishing among these grounds is irrelevant to the tally. All that matters is whether *any* basis exists for not counting a signature.

We can visualize this point in symbolic terms. At the most general level,

$$V = T - I \qquad [11]$$

where

V = the number of valid signatures;
T = the total number of signatures submitted; and
I = the number of invalid signatures.

In other words, the number of valid signatures is simply the total number of signatures less those that are invalid. But the number of invalid signatures (I) is the sum of all signatures that are invalid for one or more reasons. For example, if signatures can be invalidated for any of three reasons, we can calculate the number of invalid signatures as follows:

$$I = I_1 + I_2 + I_3 - (B_{12} + B_{13} + B_{23} - B_{123}) \qquad [12]$$

---

[39]*See, e.g.*, Clark v. Pima County Bd. of Supervisors, 624 P.2d 871, 873 (Ariz. 1981); Whitman v. Moore, 125 P.2d 445, 455 (Ariz. 1942); CAL. ELEC. CODE § 3516(a) (West 1977); DEL. CODE ANN. tit. 15, § 3002(e)(5) (1981).

[40]This problem can arise because the signer either failed to provide this information or moved to a new address without notifying election officials. *See, e.g.*, Energy Fuels Nuclear, Inc. v. Coconino County, 766 P.2d 83, 86 (Ariz. 1988); Assembly of Cal. v. Deukmejian, 639 P.2d 939, 945–47 (Cal.), *cert. denied*, 456 U.S. 941 (1982); Yes to Stop Callaway Comm. v. Kirkpatrick, 685 S.W.2d 209, 211 (Mo. Ct. App. 1984); Dawson v. Meier, 78 N.W.2d 420, 426–27 (N.D. 1956); State *ex rel.* Corrigan v. Perk, 249 N.E.2d 525, 526 (Ohio), *appeal dismissed sub nom.* Perk v. Ohio, 396 U.S. 113 (1969); Oklahomans for Modern Alcoholic Beverage Control v. Shelton, 501 P.2d 1089, 1093 (Okla. 1972); State *ex rel.* Wise v. Judd, 655 S.W.2d 952, 955 (Tenn. 1983); CAL. ELEC. CODE § 3516(c) (West 1977); TEX. ELEC. CODE ANN. § 141.063(2)(A) (Vernon 1986).

[41]*See, e.g.*, WASH. REV. CODE ANN. § 29.79.200 (Supp. 1993). Discovering duplicate signatures is a troublesome problem that most sampling systems do not address. One notable exception is Oregon. *See* OR. ADMIN. R. 165-14-030 app. A (1986). Resolving this problem is beyond the scope of the present article.

[42]*See, e.g.*, Segars v. Bramlett, 265 S.E.2d 279, 281 (Ga. 1980); McCarthy v. Secretary of the Commonwealth, 359 N.E.2d 291, 302 n.19, 303 n.22 (Mass. 1977); *In re* Initiative Petition No. 317, 648 P.2d 1207, 1215 (Okla. 1982); ALASKA STAT. § 29.26.280(c) (1992); IDAHO CODE § 34-1706(1)(b)(ii), (2)(a)(ii) (1981); KAN. STAT. ANN. §§ 25-4310, 25-4324 (Supp. 1992); LA. REV. STAT. ANN. §§ 18:3(A)(5), 18:465(D) (West 1979 & Supp. 1993).

[43]*See, e.g.*, ARIZ. REV. STAT. ANN. § 19-112(B) (1990); ILL. COMP. STAT. ANN. ch. 10 § 5/28-10 (Smith-Hurd 1993); MO. ANN. STAT. § 116.060 (Vernon Supp. 1992); MONT. CODE ANN. § 13-10-501(5) (1991).

[44]*See, e.g.*, CAL. ELEC. CODE § 3519 (West 1977 & Supp. 1993); NEV. REV. STAT. § 293.128(2) (1987).

[45]*See, e.g.*, CAL. ELEC. CODE §§ 3520(a), 27212 (West 1977, 1989, & Supp. 1993); CAL. PUB. UTIL. CODE § 12815.7 (WEST SUPP. 1993).

where

I = the total number of invalid signatures;
I_1 = the number of signatures that are invalid for reason 1;
I_2 = the number of signatures that are invalid for reason 2;
I_3 = the number of signatures that are invalid for reason 3;
B_{12} = the number of signatures that are invalid for both reason 1 and reason 2;
B_{13} = the number of signatures that are invalid for both reason 1 and reason 3;
B_{23} = the number of signatures that are invalid for both reason 2 and reason 3; and
B_{123} = the number of signatures that are invalid for all of reasons 1, 2, and 3.

Analogous formulas would permit us to calculate the number of invalid signatures when there is a larger number of grounds for invalidity.[46]

It follows that we can redefine the number of valid signatures as the total number of signatures less the number of signatures that are invalid for one or more reasons. With three possible reasons for invalidity, we would proceed as follows:

$$V = T - [I_1 + I_2 + I_3 - (B_{12} + B_{13} + B_{23} - B_{123})].^{47} \qquad [13]$$

Because we do not care why any particular signature is invalid, our counting mechanism should ignore the specific grounds for invalidity. A cursory glance at equation 13 suggests how complex it would be to calculate the number of valid signatures through a process that distinguishes among the different kinds of invalid signatures. Equation 11 is algebraically equivalent to equation 13. Its greater simplicity allows us to make the count faster and more accurately.

This principle also applies to sampling statutes. A properly selected random sample will enable us to estimate with measurable accuracy the number

[46]We can calculate the number of invalid signatures when n grounds for invalidity exist according to the general formula:

$$I = \sum_{i=1}^{n} I_i - \sum_{i_i=1}^{n} \sum_{i_2=1}^{i_i} I_{i_i i_2} + \sum_{i_l=l}^{n} \sum_{i_2=1}^{i_l} \sum_{i_3=1}^{i_2} I_{i_l i_2 i_3} - \ldots \pm I_{i_l i_2 \ldots i_n}$$

where

$\sum_{i=1}^{n}$ = the sum of all signatures that are invalid for reasons 1 through n; and the remaining terms adjust for the possible combinations of overlapping reasons 1 through n for finding signatures invalid.

[47]Analogously, the number of valid signatures when n grounds for invalidity exist is given by the general formula:

$$V = T - \left[\sum_{i=1}^{n} I_i - \sum_{i_l=1}^{n} I_{i_l i_2} + \sum_{i_l=1}^{n} \sum_{i_2=1}^{i_l} \sum_{i_3=1}^{i_2} I_{i_l i_2 i_3} - \ldots \pm I_{i_l i_2 \ldots i_n} \right]$$

of valid and invalid signatures in the total pool. To be sure, the sampling procedure might not give as good data about each kind of invalid signature, but that does not affect the accuracy of the overall count. And, of course, the overall count is the only number that matters.

We can estimate the number of valid signatures using a properly selected random sample as follows:

$$V_e = (v/t)(T) \qquad [\mathbf{14}]$$

where

V_e = the estimated number of valid signatures in the total pool;
v = the number of valid signatures in the sample;
t = the total number of signatures in the sample; and
T = the total number of signatures submitted.

In other words, we can estimate the number of valid signatures by determining the proportion of valid signatures in the sample and taking a "like percentage" of the total pool.

We can verify this statement by applying it to the examples we considered earlier. In our paradigm hypothetical,[48] for instance, we would first calculate the number of valid signatures in the sample using equation 2:

$$\begin{aligned} v &= t - (c + s - b) \\ &= 1{,}000 - (200 + 300 - 50) \\ &= 1{,}000 - 450 \\ &= 550. \end{aligned}$$

Then we would go to equation 14 and find:

$$\begin{aligned} V_e &= (v/t)(T) \\ &= (550/1{,}000)(20{,}000) \\ &= (.55)(20{,}000) \\ &= 11{,}000. \end{aligned}$$

This figure is, of course, precisely correct. Equation 14 yields an equally accurate estimate in the other example that we considered above.[49]

[48]The paradigm hypothetical involves: a petition containing 20,000 signatures (T); 4,000 of which were obtained by ineligible circulators (C); 6,000 of which came from ineligible signers (S); and 1,000 of which are invalid on both grounds (B). In the five percent random sample, we have: 1,000 signatures (t); 200 of which were obtained by ineligible circulators (c); 300 of which came from ineligible signers (s); and 50 of which are ineligible on both grounds (b).

[49]In this second hypothetical example, we have: a petition containing 20,000 signatures (T), 4,000 of which were obtained by ineligible circulators (C), 16,000 of which came from ineligible signers (S), and 2,000 of which are invalid on both grounds (B); the five percent random sample of 1,000 signatures (t) includes 200 that were obtained by ineligible circulators (c), 800 that came from ineligible signers (s), and 100 that are invalid on both grounds (b). The analysis here would proceed in the same fashion. First, calculate the number of valid signatures in the sample as

$$\begin{aligned} v &= t - (c + s - b) \\ &= 1{,}000 - (200 + 800 - 100) \\ &= 1{,}000 - 900 \\ &= 100. \end{aligned}$$

Various mathematically equivalent formulas of similar algebraic simplicity exist.[50] Each will generate accurate estimates of the number of valid signatures from properly selected random samples of signatures on any petition. However, the discussion thus far, illuminates how the Arizona sampling statute should have been interpreted.

III. DISCUSSION AND IMPLICATIONS

The preceding discussion suggests that the most efficient way to estimate the number of valid signatures on a petition is to select a random sample of the submitted signatures, determine how many of the sample signatures are valid (ignoring the reasons why some signatures are invalid), calculate the proportion of valid signatures in the sample, and apply that proportion to the total number of signatures submitted. The Arizona sampling statute does not follow this course. Instead, it refers specifically to two grounds for not counting signatures: the ineligibility of the circulator and the ineligibility of the signer. This fact, together with the ambiguous wording of the statute, gave rise to the inconsistent and mathematically unsound interpretations that were analyzed earlier.[51]

These problems could have been avoided by glossing the statute to reach a sensible result. The court in *City of Flagstaff v. Mangum* declined to do that, emphasizing that its interpretation "appears clearly mandated by the statute."[52] This explanation is unsatisfactory. The sampling statute, after all, was not promulgated by a specialized body acting "within its area of special expertise, at

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Then, using equation 14, project this figure:

$$\begin{aligned} V_e &= (v/t)T \\ &= (100/1{,}000)(20{,}000) \\ &= (.1)(20{,}000) \\ &= 2{,}000. \end{aligned}$$

We know that this is the correct answer. *See supra* note 16. Recall that the city's approach gave a negative estimate of the number of valid signatures in this example, whereas the court's generated an exaggerated figure.

[50]For example, we could incorporate equation 2 directly into our calculation as follows:

$$V_e = T - (i/t)T$$

where

V_e = the estimated number of valid signatures in the total pool;
T = the total number of signatures submitted;
i = the number of invalid signatures in the sample; and
t = the total number of signatures in the sample.

[51]The statute's structural problems arose from its history. As originally enacted, § 19-121.04 made no provision for sampling. Rather, it made clear that signatures obtained by ineligible circulators as well as signatures given by ineligible signers were invalid. Act of May 14, 1973, ch. 159, § 7, 1973 Ariz. Sess. Laws 1559, 1567. Language authorizing sampling as part of the signature-verification process was inserted in 1977. Act of May 31, 1977, ch. 135, §§ 7–8, 1977 Ariz. Sess. Laws 617, 623–25. The recent revision of the sampling statute, *see supra* note 23, appears to embody a mathematically sound approach for estimating the number of valid signatures. The new provision is, however, more complex than the approach advocated here.

[52]793 P.2d 548, 557 (Ariz. 1990).

the frontiers of science.''[53] Nor was this an instance of legislation containing a possibly mistaken, but nevertheless plausible, provision.[54] In such situations, upholding the literal language of the statute might have been appropriate. Yet this issue involved the use of elementary algebra. In that sense, it was reminiscent of the apparently apocryphal legislative proposal to set π equal to 3.[55]

There are two difficulties with the court's reasoning. First, the judicial interpretation of the sampling statute was not itself faithful to the literal language of the statute.[56] Second, the court has previously refused to construe statutes in a blindly literal fashion if doing so would produce an absurd result.[57] Accordingly, it is difficult to understand why the court failed to interpret the statute so as to produce mathematically sound estimates of the number of valid signatures.[58]

The problem cannot have arisen from the use of words rather than mathematical symbols to explain the necessary calculations, because the pioneering work in algebra occurred centuries before algebraic notation was developed.[59] The difficulty goes deeper. Lawyers and judges are rarely comfortable when forced to consider quantitative matters.[60] Courts typically seek to avoid immersing themselves in the validity of mathematical analyses.[61] Even when the judiciary ventures into the statistical thicket, its performance often has been superficial at best.[62] Unfortu-

[53]Baltimore Gas & Elec. Co. v. Natural Resources Defense Council Inc., 462 U.S. 87, 103 (1983).

[54]*E.g.*, United States v. Locke, 471 U.S. 84, 93-96 (1985).

[55]For an account of an actual, and even more confused, attempt to legislate the value of π, see PETR BECKMANN, THE HISTORY OF π 174–77 (1971).

[56]*See* 793 P.2d at 556 (rejecting city's approach because, although literally faithful to statutory language, it ''gravely distorts'' estimates to detriment of petition proponents).

[57]*See, e.g.*, Arnold Constr. Co. v. Arizona Bd. of Regents, 512 P.2d 1229, 1232 (Ariz. 1973); Garrison v. Luke, 78 P.2d 1120, 1122 (Ariz. 1938); Keller v. State, 47 P.2d 442, 447 (Ariz. 1935); City of Phoenix v. Superior Court, 696 P.2d 724, 729 (Ariz. Ct. App. 1985); Knoell Bros. Constr. Inc. v. State, 644 P.2d 905, 908 (Ariz. Ct. App. 1982).

[58]The court's unwillingness to reach a mathematically sensible result is especially distressing because it already had determined that the petition at issue was untimely filed and hence null and void. 793 P.2d at 551-53. Accordingly, it was not necessary to interpret the sampling statute in order to resolve the case. Nevertheless, the court chose to do so because the meaning of that statute was a ''significant question[] of public importance'' which was ''likely to recur.'' *Id.* at 553.

[59]The Arabs, who played an important role in the development of algebra, did all of their work in verbal rather than symbolic terms. WILLIAM DUNHAM, JOURNEY THROUGH GENIUS 131 (1990). So, too, did Euclid in his exposition of what is now known as geometric algebra. *Id.* at 61. Algebraic notation did not appear until the late sixteenth century. *Id.* at 156.

[60]To be sure, fixing damages in tort and contract cases requires calculation, a task that courts have performed unblinkingly for centuries. Statistical evidence is central to much antitrust and civil rights litigation. *See, e.g.*, Bazemore v. Friday, 478 U.S. 385, 401-03 and n.14 (1986) (Brennan, J., joined by all other members of the Court, concurring in part) (analyzing multiple regression analyses in employment discrimination case). And almost all of tax law involves numbers, a fact that terrifies many students who take the basic course in that subject.

[61]*See, e.g.*, McCleskey v. Kemp, 481 U.S. 279, 291-313 and n.7 (1987) (assuming that an empirical study of racial differentials in the imposition of the death penalty was statistically valid but finding the study not probative of the constitutional claims at issue).

[62]*See, e.g.*, Swain v. Alabama, 380 U.S. 202, 205-06 (1965) (evaluating a claim of racial discrimination in jury selection through comparison of actual population with observed venire rather than through comparison of observed venire with the expected venire based upon chance);

nately, innumeracy in the legal system reflects broad mathematical indifference or incompetence in society at large.[63]

There are several other lessons in this melancholy episode. This problem arose from the legislature's attempt to enact a mathematical formula when that task could have been vested in the administrative agency responsible for implementing the sampling statute. By seeking to constrain the agency's discretion, the legislature was taking seriously the delegation doctrine, which has been more honored in the breach than in the observance over the years.[64] Perhaps it would have sufficed for the legislature to direct the agency to utilize a scientifically acceptable form of random sampling in estimating the number of valid signatures and left the details to the administrators.[65]

More significant, the surprising difficulty of the seemingly mundane task of counting signatures ought to give pause to those who believe that we can readily distinguish between those disputes that are appropriate for judicial resolution and those that are not. The former category may be referred to as bipolar disputes, which typically involve a small number of parties with clearly defined interests, whereas the latter category includes what have been called polycentric problems, which typically implicate multiple parties or interests.[66] Polycentric disputes have been analogized to a spider's web in which changes in the force applied to one strand will redistribute the tension on all strands in complicated and unpredictable fashion.[67]

On closer inspection, however, this distinction breaks down. Some bipolar disputes clearly implicate interests beyond those of the parties to a lawsuit. For example, common law rules of contract, tort, and property are intended to influence the behavior of numerous nonparties. Similarly, no one suggests

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see Michael O. Finkelstein, *The Application of Statistical Decision Theory to the Jury Discrimination Cases*, 80 HARV. L. REV. 338, 353–63 (1966); *see also* Montana v. United States Dep't of Commerce, 775 F. Supp. 1358, 1362 (D. Mont.) (three-judge court) (acknowledging "mathematical impossibility" of applying "one person, one vote" standard to process for allocating seats in House of Representatives among states but insisting upon applying that standard anyway), *rev'd*, 112 S. Ct. 1415 (1992). *But see* Castaneda v. Partida, 430 U.S. 482, 496–97 n.17 (1977) (applying the binomial theorem to analyze a jury-discrimination claim).

[63] *See generally* PAULOS, *supra* note 9. Consider the following lament by an atmospheric physicist: "My students were 'nonscience' majors, although 'anti-science' would perhaps be closer to the mark. . . . The merest whiff of an equation would stampede them to the dean's office to complain about cruel and unusual punishment." CRAIG F. BOHREN, CLOUDS IN A GLASS OF BEER ix (1987).

[64] *See, e.g.*, Mistretta v. United States, 488 U.S. 361 (1989); Yakus v. United States, 321 U.S. 414 (1944); J.W. Hampton, Jr. & Co. v. United States, 276 U.S. 394 (1928); Field v. Clark, 143 U.S. 649 (1892); Amalgamated Meat Cutters v. Connally, 337 F. Supp. 737 (D.D.C. 1971) (three-judge court).

[65] *Cf.* Jerry L. Mashaw, *Prodelegation: Why Administrators Should Make Political Decisions*, 1 J.L. ECON. & ORG. 81, 85–91 (1985) (suggesting that agency officials are often able to make more rational policy choices than are legislators). *But see supra* notes 13–17 and accompanying text (observing that administrators have not always drawn samples according to generally accepted scientific principles).

[66] *See supra* note 1 and accompanying text.

[67] Fuller, *supra* note 1, at 395.

that criminal proceedings are inappropriate for judicial resolution even though several of the traditional justifications for imposing punishment are directed to deterring others from engaging in antisocial conduct or protecting strangers from exploitation by third parties.[68]

At the same time, some polycentric disputes might be suitable for adjudication. Consider one example given by Lon Fuller: a baseball manager's decisions concerning his lineup and strategic moves during the course of a game.[69] Even if Fuller is correct that reaching those decisions implicates a form of reasoning that is incompatible with the normal work of courts, a slight variation will illustrate a possible, albeit limited, role for adjudication. Suppose that the manager refuses to use a player out of racial, ethnic, or religious prejudice. Such a decision might well violate applicable civil rights laws and would be actionable as a form of employment discrimination.[70] By the same token, many complex matters that implicate numerous parties and diverse interests have traditionally been handled by common law courts. Among these are the administration of trusts and estates and the management of the affairs of debtors who seek relief in bankruptcy.[71] In addition, some European specialized courts have found it extremely difficult to draw clear lines between claims by individual workers, over which the tribunals have jurisdiction, and collective claims, over which they do not.[72]

To say that the distinction between bipolar and polycentric disputes is often difficult to identify does not mean that all such disputes are indistinguishable from each other. The imprecision of these categories means only that the distinction may be more useful as a means of facilitating discussion about the optimal way to resolve particular disputes than as a principle for excluding entire classes of claims from the judicial process.[73]

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[68]*See* SANFORD H. KADISH & STEPHEN J. SCHULHOFER, CRIMINAL LAW AND ITS PROCESSES 149–53, 160–65 (5th ed. 1989). The Supreme Court's acrimonious dispute over the role of victim-impact statements in capital cases provides another illustration of the extent to which multiple interests affect a criminal trial in which there are only two nominal parties. *See* Payne v. Tennessee, 111 S. Ct. 2597 (1991), *overruling* Gathers v. South Carolina, 490 U.S. 805 (1989), *and* Booth v. Maryland, 482 U.S. 496 (1987).

[69]Fuller, *supra* note 1, at 398, 399–400.

[70]For example, federal and state antidiscrimination agencies mediated a complaint against the Boston Red Sox after the team fired Tommy Harper, a black coach. *Harper Case Settled*, N.Y. TIMES, Dec. 6, 1986, § 1, at 50; *see also* David Margolick, *Boston Case Revives Past and Passions*, N.Y. TIMES, Mar. 23, 1986, § 5, at 1 (discussing background to Harper dispute and other allegations of racism against Boston baseball team). Earlier, officials of Major League Baseball threatened to take disciplinary action against any player or team that refused to play against the Brooklyn Dodgers when Jackie Robinson broke the unwritten color barrier in 1947. JULES TYGIEL, BASEBALL'S GREAT EXPERIMENT 184–88 (1983).

[71]Eisenberg & Yeazell, *supra* note 2, at 482–86.

[72]Bob Hepple, *Labour Courts: Some Comparative Perspectives*, 41 CURRENT LEGAL PROBS. 169, 174 (1988).

[73]*Cf.* Peter L. Strauss, *The Place of Agencies in Government: Separation of Powers and the Fourth Branch*, 84 COLUM. L. REV. 573, 625 (1984) (suggesting that functional approach to separation of powers problems may be "more effective as a means of organizing debate than as a rule for deciding cases").

IV. CONCLUSION

Many lawyers take an almost perverse pride in their lack of mathematical sophistication. Sometimes this innumeracy serves as an obstacle to sensible thought about legal problems. This article has sought to demonstrate that simple formulas about institutional competence can create real mischief.